

Edge Detection Using Young's Reaction-Diffusion Cellular Automaton with Nonlinear Function

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Extended abstract

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This abstract introduces a technique to convert the two-dimensional cellular automaton (CA) pattern formation model proposed by D. A. Young [4] into an edge detector with nonlinear functions.

Our previous study proposed an extended version of Young's model [3], and reported that the model enables not only the formation of spotted and labyrinthine patterns, but also various types of image processing, such as edge detection and skeletonization, by adjusting the parameters [2]. The model is defined by 5-tuple $(\mathcal{L}, \mathcal{T}, \mathcal{S}, \mathcal{N}, \phi)$, consisting of a two-dimensional lattice $\mathcal{L} = \mathbb{Z}^2$, a timeline $\mathcal{T} = \mathbb{Z}$, a state set $\mathcal{S} = \{0, 1\}$, a neighborhood $\mathcal{N} \subset \mathcal{L}$, a local rule ϕ . The boundary condition in the lattice $\mathcal{L}$ is periodic in this paper. Let $c^t \in \mathcal{S}^{\mathcal{L}}$ be the configuration of cells at time $t \in \mathcal{T}$, and let $c^t(\mathbf{i})$ denote the cell state at lattice point $\mathbf{i} \in \mathcal{L}$. The neighborhood $\mathcal{N} \subset \mathcal{L}$ of the model is composed of three regions: the central point $\{(0, 0)\}$, an activation area $\mathcal{N}_A$, and an inhibition area $\mathcal{N}_I$, given by $\mathcal{N}_A = \{\mathbf{i} \in \mathbb{Z}^2 \mid 0 < \|\mathbf{i}\|_{\infty} \leq r_A\}$, $\mathcal{N}_I = \{\mathbf{i} \in \mathbb{Z}^2 \mid r_A < \|\mathbf{i}\|_{\infty} \leq r_I\}$, respectively. The parameters $r_A, r_I \in \mathbb{N}$ determine the sizes of the two neighborhoods, where $r_A < r_I$. We fixed $r_A = 5$ and $r_I = 9$ throughout this paper. Here, $\|\cdot\|_{\infty}$ denotes the L_{∞} norm. The states of cells are updated synchronously in discrete time according to the local rule,

$$c^{t+1}(\mathbf{i}) = \phi(c^t(\mathbf{i}), \ell_{A_i}^t, \ell_{I_i}^t) = \begin{cases} 0 & \text{if } m(\ell_{A_i}^t, \ell_{I_i}^t) < 0, \\ c^t(\mathbf{i}) & \text{if } m(\ell_{A_i}^t, \ell_{I_i}^t) = 0, \\ 1 & \text{if } m(\ell_{A_i}^t, \ell_{I_i}^t) > 0, \end{cases} \quad (1)$$

where $\ell_{A_i}^t$ and $\ell_{I_i}^t$ are the number of cells in state 1 within $\mathcal{N}_A$ and $\mathcal{N}_I$ around the lattice point $\mathbf{i}$, respectively. Their respective ranges are $L_A = \{0, 1, \dots, |\mathcal{N}_A|\} \subset \mathbb{Z}$ and $L_I = \{0, 1, \dots, |\mathcal{N}_I|\} \subset \mathbb{Z}$, where $|\cdot|$ represents the cardinality of a set. The function m , defined on $L_A \times L_I$ to the real numbers, is called the reader function. In this paper, we define this function as

$$m(\ell_{A_i}^t, \ell_{I_i}^t) = \begin{cases} 1 & \text{if } (\ell_{A_i}^t, \ell_{I_i}^t) \in Q, \\ \ell_{A_i}^t - w\ell_{I_i}^t - b & \text{otherwise,} \end{cases} \quad (2)$$

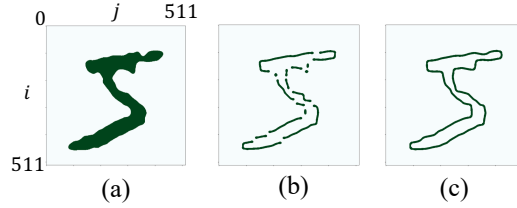


Fig. 1. (a) A handwritten digit obtained from MNIST [1]. Pixel values are binarized using a threshold. Light and dark colors indicate state 0 and state 1, respectively. (b, c) Images obtained after 200 updates of (a) by CA. Parameters: (b) $w = 0.5$, $b = 20$, $Q = \emptyset$; (c) $w = 0.5$, $b = 20$, $Q = \{(\ell_{A_i}^t, \ell_{I_i}^t) \in L_A \times L_I \mid 63 \leq \ell_{A_i}^t < 123 \text{ and } 118 \leq \ell_{I_i}^t < 178\}$.

where the parameter set $Q \subseteq L_A \times L_I$ and $w, b \in \mathbb{R}$ can be set arbitrarily. Notably, when $Q = \emptyset$ and $b = 0$, the local rule ϕ of the extended Young's model are equivalent to that of the original Young's model. A previous study [2] investigated the case where m is a linear function ($Q = \emptyset$ and $b \neq 0$), and reported that the following three conditions are necessary for edge detection: (i) $m(\ell_{A_i}^t, \ell_{I_i}^t) \leq 0$ at $(\ell_{A_i}^t, \ell_{I_i}^t) = (0, 0)$, (ii) $m(\ell_{A_i}^t, \ell_{I_i}^t) \geq 0$ when $0 \ll \ell_{A_i}^t \ll |\mathcal{N}_A|$ and $0 \ll \ell_{I_i}^t \ll |\mathcal{N}_I|$, and (iii) $m(\ell_{A_i}^t, \ell_{I_i}^t) < 0$ at $(\ell_{A_i}^t, \ell_{I_i}^t) = (|\mathcal{N}_A|, |\mathcal{N}_I|)$. Since the function m in condition (ii) has a different sign from (i) and (iii), it is difficult to satisfy them simultaneously when m is a linear function. However, using a nonlinear function makes it straightforward to satisfy these conditions.

We performed edge detection on a handwritten digit image (Fig. 1(a)) using the extended Young's model. Each pixel of the image was treated as a cell to define the initial configuration in CA. Figure 1(b) shows the image evolved using a linear reader function that satisfies only conditions (i) and (iii). In Fig. 1(c), a nonlinear function created by adding region Q to the reader function of (b) produces clear edges. This improvement can be attributed to the Q , which allows the function to satisfy condition (ii). These results demonstrate that introducing nonlinearity into the reader function can enable edge detection in certain cases. Although we reported the results for a specific set of parameter values in this paper, introducing region Q enables edge detection with other parameter values as well. In future work, we plan to systematically vary Q to identify the edge-detectable parameter conditions in the extended Young's model.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

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